Noise Figure and Homodyne Noise Figure

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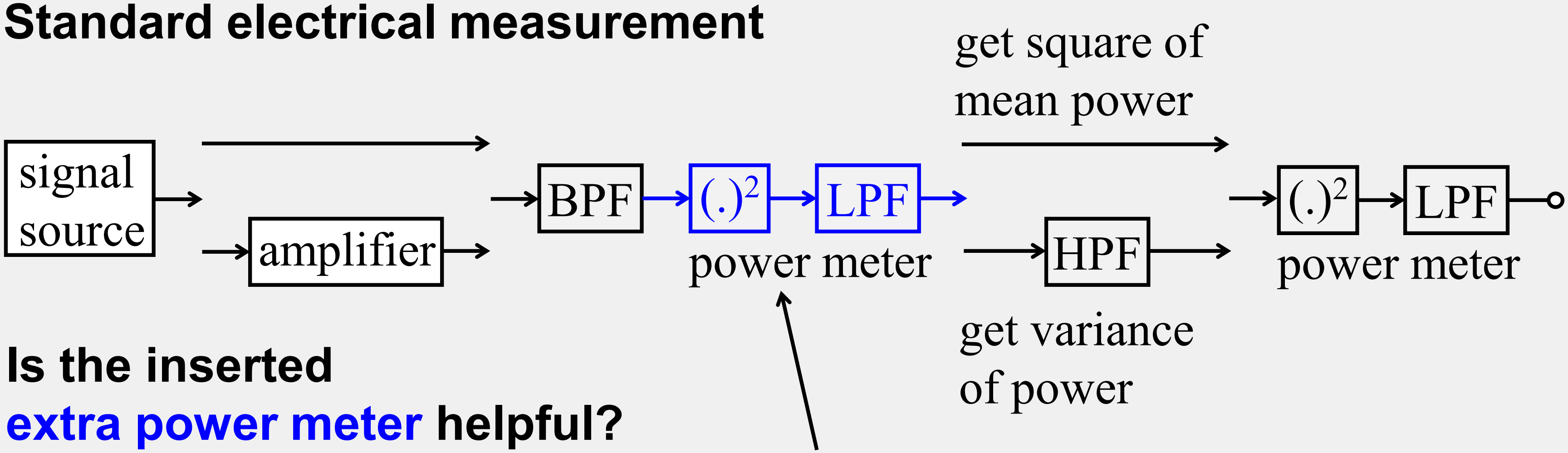
How to determine noise and gain properties of amplifier

Standard electrical measurement



Is the inserted
extra power meter helpful?

Probably not. Now this is a photodiode and we are talking about optical signals!

$$G = e^{(a-b)t} = e^{(a-b)z/v_g}$$ gain

$$\mu = n_{sp}(G-1)$$ mean number of detectable output noise photons per mode

$$n_{sp} = \frac{a}{a-b}$$ spontaneous emission factor

$$P_{n,out}\tau = \mu hf = G\tilde{\mu} hf$$ mean output noise energy per mode

F_e, gain, loss, power must be redefined if F_{pnf} is valid NF!

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_s P_{n,out}}{P_n P_{s,out}} = \frac{\text{noise gain}}{\boxed{\text{signal gain}}}$$

Science is **systematic** and **exact** and **does not tolerate contradictions**! Unit definitions **must not** depend on measurement method or f! F_{pnf} implies:

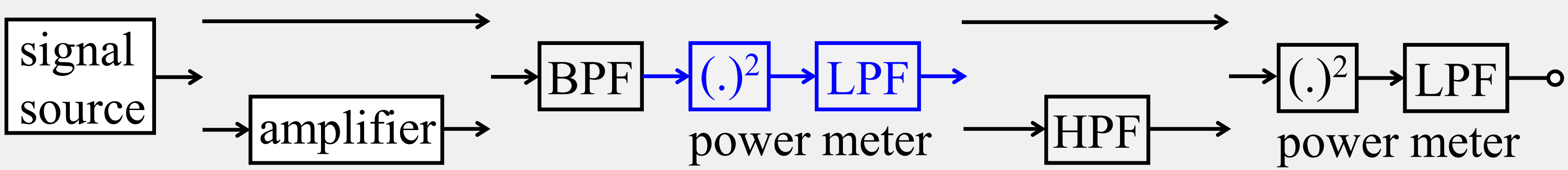
$$\frac{G^2\langle n\rangle^2}{\langle n\rangle^2} = G^2$$ **is** the signal „gain"! ⇔ **No more (optical) dB are allowed; „gain" must be given in „electrical" dB! ⇔ Fiber loss @ 1550 nm is no longer 0.2 dB/km; „loss" must be stated as 0.4 dB/km! ϟ**

⇔ A thermal power meter can replace the photodiode and then allows going to low f. There also a Schottky detector can be used.

⇔ Any electrical or optical amplifier with 20 dB gain has 40 dB „gain"! ϟ ⇔ All „powers" (optical, electrical, thermal, mechanical) must be ~ squared powers, since all powers can be converted into thermal power and compared as such! ϟ

$$SNR_{pnf,in} = \frac{\langle n\rangle^2}{\langle n\rangle}$$

$$SNR_{pnf,out} = \frac{G^2\langle n\rangle^2}{G\langle n\rangle + n_{sp}\left(2(G-1)G\langle n\rangle + \ldots\right)}$$



Problem introduction

- **Electrical** noise figure (NF) is standardized since many decades.
- Traditional **optical** noise figure F_{pnf} was defined in 1990ies, for optical direct detection receivers (DD RX). Problematic aspects, in **conflict** with electrical NF:
  - Optical signals have in-phase and quadrature components, like electrical signals. But an optical DD RX **suppresses** phase information.
  - „Power“ in signal-to-noise (SNR) ratio calculation is ~ square of photocurrent in optical DD RX. Photocurrent is ~ optical power ~ **square** of field amplitude. SNR „power“ is ~ 4th power of field amplitude ~ **square** of power. ϟ
    Conflict with ~150 years of science: $P = U^2/R$, not $P \sim U^4$.

  $\Rightarrow$ NF = 2 for ideal optical amplifier, whereas NF = 1 for ideal electrical amplifier.
  - **Noise happens on a field basis. Looking at the power is insufficient!** ϟ
  - Ideal DD RX for intensity modulation without / with ideal optical amplifier needs 10 / 38 photoelectrons/bit for bit error ratio = 10^{-9}. Ideal DD RX for differential phase shift keying: 20 / 20 photoelectrons/bit. **Where is NF = 2?**
  - Optical: **Non**linear DD RX; **non**-Gaussian noise; amplifier NF depends on power and bandwidths. <u>Electrical: Linear RX; Gaussian noise; constant NF.</u>
  - Unification of all prior optical NF with electrical NF is inconsistent, contradictory.

Fields in coherent optical I&Q receiver

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

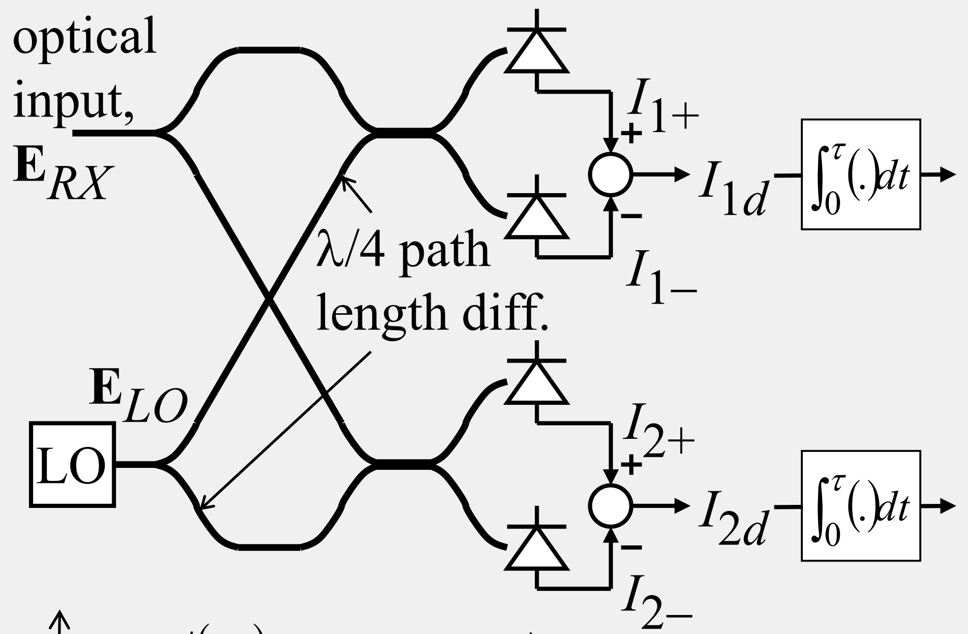
$$P =: |\mathbf{E}|^2 \qquad I = RP = e/(hf)\cdot P$$

Power (for simplicity) Photocurrent

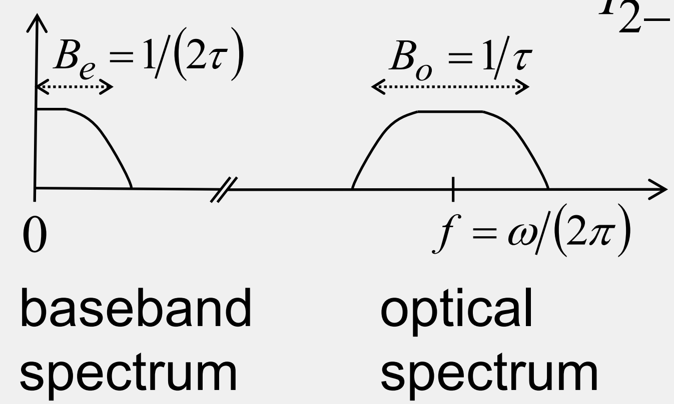
v_1, v_2 independent zero-mean Gaussian

$$\langle v_1^2 \rangle = \langle v_2^2 \rangle = \sigma^2 = 1$$

$\mathbf{e}_1$ normalized field (polarization) vector



Optical signal is downconverted into baseband. Local oscillator (LO) is a strong unmodulated laser with (essentially) the same frequency as the received signal.



2 available quadratures

Baseband I&Q receiver is not mandatory!

Heterodyne receiver with image rejection filter gives the same results!

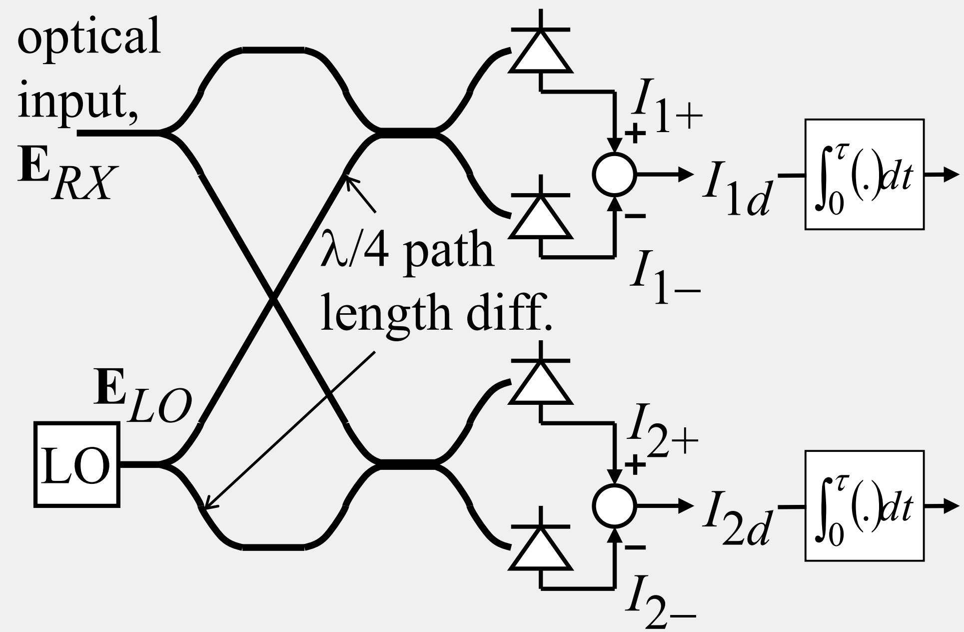
Photocurrents in coherent optical I&Q receiver ...

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$P =: |\mathbf{E}|^2 \qquad I = RP = e/(hf)\cdot P$$

(In practice, optical frequencies of signal and unmodulated local oscillator may differ a bit, causing the complex plane of I_{1d} and I_{2d} to rotate at the difference frequency.)



4 detected photocurrents

$$I_{1\pm} = R\left|\pm \mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + P_{LO}\right)$$

$$I_{2\pm} = R\left|\pm \mathbf{E}_{RX}/2 + j\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + P_{LO}\right)$$

...and their differences and sums

$$I_{1d} = I_{1+} - I_{1-} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

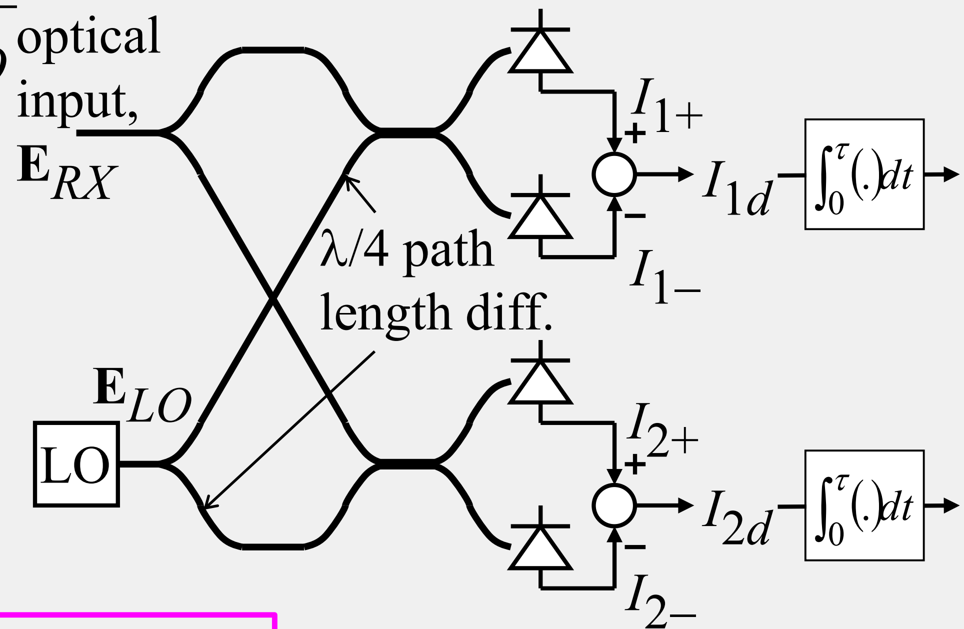
$$I_{2d} = I_{2+} - I_{2-} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = I_{1+} + I_{1-} = RP_{LO}/2$$

$$I_{2s} = I_{2+} + I_{2-} = RP_{LO}/2$$

Differences and sums of photocurrents

$\boxed{\text{Neglect for } P_{LO} \to \infty}$



$\boxed{\text{Cancel in subtraction}}$

4 detected photocurrents

$$I_{1\pm} = R\left|\pm\mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(\cancel{G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right)} \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + \cancel{P_{LO}}\right)$$

$$I_{2\pm} = R\left|\pm\mathbf{E}_{RX}/2 + j\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(\cancel{G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right)} \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + \cancel{P_{LO}}\right)$$

SNR in coherent optical I&Q receiver

$$I_{1d} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

$$I_{2d} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = RP_{LO}/2$$

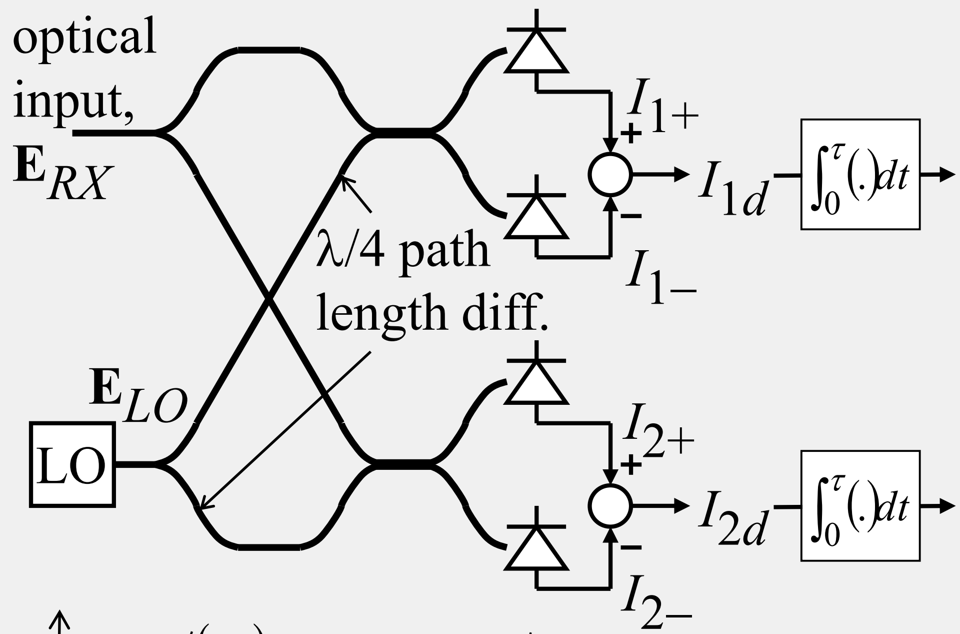
$$I_{2s} = RP_{LO}/2$$

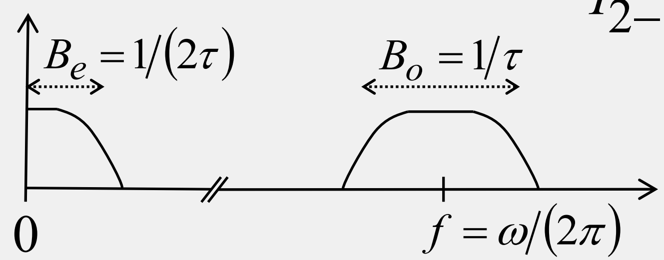
Pure Gaussian PDFs of interference + field noises!

Shot noise PSD: $2eI_{1s}\ ,\ 2eI_{2s}$

Optical bandwidth: $B_o = 2B_e = 1/\tau$

Equivalent amplifier input noise PSD per mode: $\tilde{\mu}hf = P_n/B_o$





For SNR calculation take either noise in 1 mode or (like I do it) in 1 quadrature! (Factor 2 cancels in NF calculation.)

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO}\, G\tilde{\mu}hfB_o/2 + eRP_{LO}B_e} = \frac{P_S\tau}{\left(\tilde{\mu} + 1/G\right)hf/2}$$

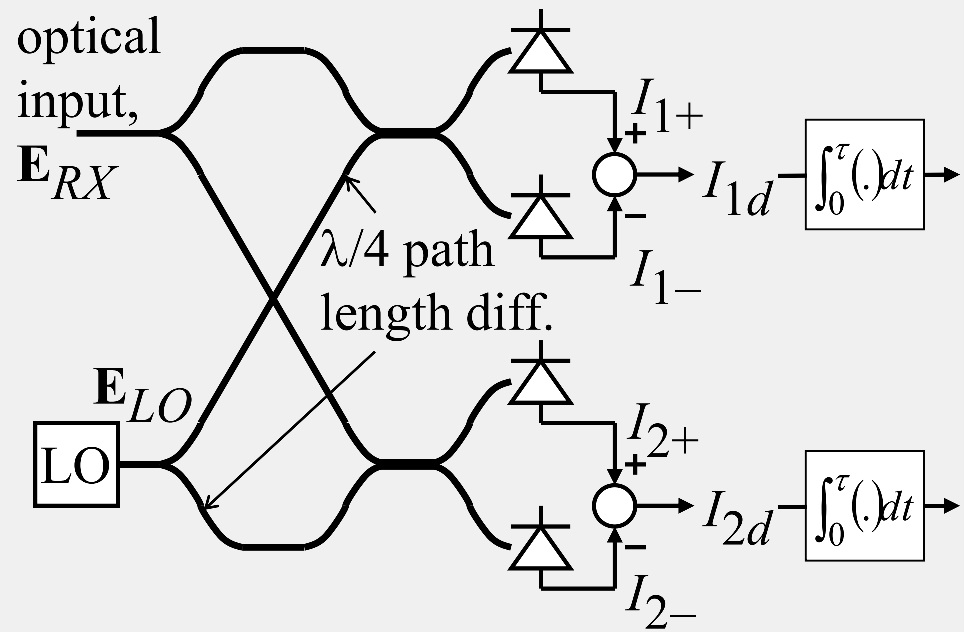
Optical I&Q noise figure (or heterodyne with image rej.)

$$SNR_{o,IQ,out} = \frac{P_S \tau}{(\tilde{\mu} + 1/G) hf/2}$$

No amplifier, $G = 1, \quad \tilde{\mu} = 0$:

$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2}$$

$$\frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} =$$



$$\boxed{F_{o,IQ} = \tilde{\mu} + 1/G = n_{sp}(1 - 1/G) + 1/G}$$

$$= 1 + (n_{sp} - 1)(1 - 1/G) \geq 1$$

$F_{o,IQ}$ obeys the usual electrical NF definition, is SNR degradation factor; powers ~ squares of amplitudes; 2 available quadratures; linearity; ideal NF = 1; pure Gaussian noise!

$$F_{o,IQ} = (F_{pnf} - 1/G)/2 + 1/G$$

$$F_{pnf} = 2(F_{o,IQ} - 1/G) + 1/G$$

$$\boxed{F_{o,IQ} \approx F_{pnf}/2}$$

for large G

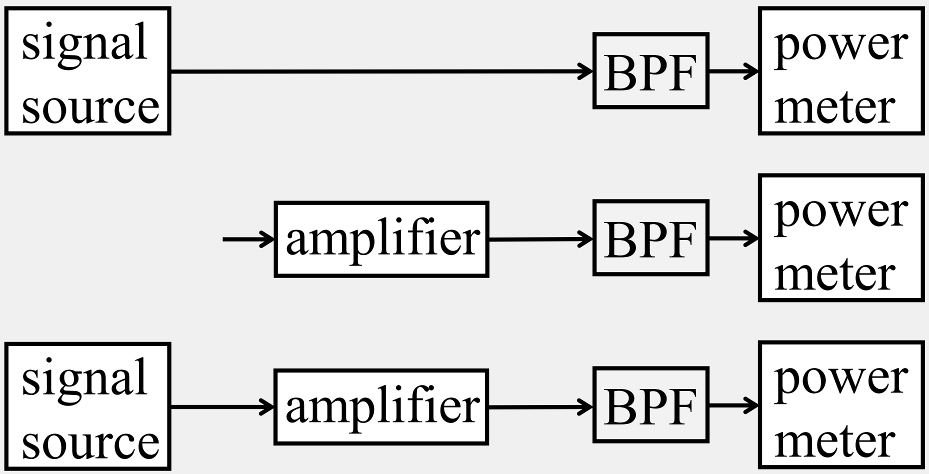
Conversion formulas

≈ 3 dB difference

Measure optical I&Q noise figure with power meter

Optical amplifier must be loaded with extra optical signal power at other times/frequencies/polarization in order to keep G, $\tilde{\mu}$ constant.

Usually there are $p=2$ polarization modes. $p=1$ requires inserted polarizer.



P_s

$P_{Gpn} = Gp\tilde{\mu}hfB_o$

$P_{G(s+pn)} = GP_s + Gp\tilde{\mu}hfB_o$

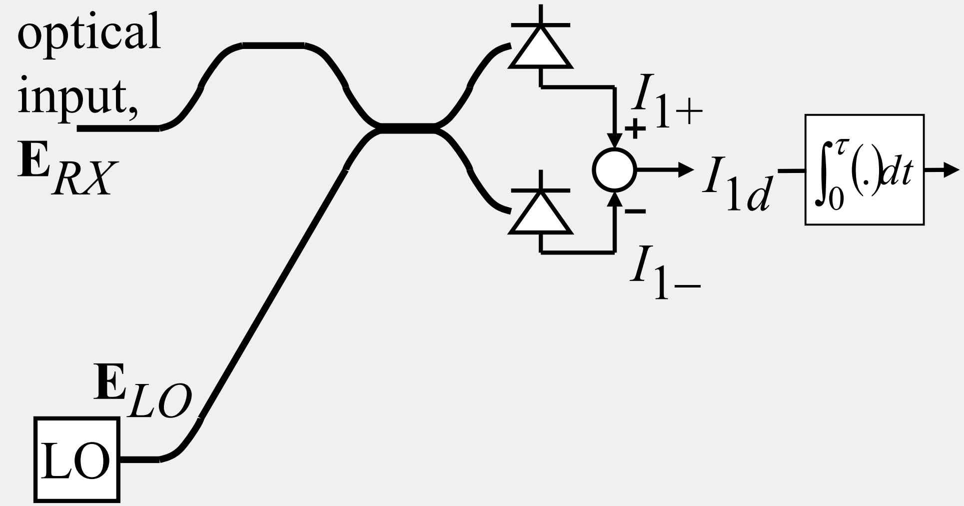
$$G = \frac{P_{G(s+pn)} - P_{Gpn}}{P_s} \qquad \tilde{\mu} = \frac{P_{Gpn}}{pGhfB_o}$$

$$\boxed{F_{o,IQ} = \tilde{\mu} + 1/G}$$

$F_{o,IQ}$ and all other optical NF can be determined from simple optical power measurements.

Optical I noise figure (true homodyne)

In such cases, phase locking is required between signal and LO or detector!
No power splitting $\Rightarrow$ In equations multiply each of $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ by 2.



$$\boxed{F_{o,I} = 2\tilde{\mu} + 1/G} \qquad (= F_{fas} = F_{pnf})$$

$$= 1 + (2n_{sp} - 1)(1 - 1/G) \geq 1$$

$F_{o,I}$ is similar to $F_{o,IQ}$ and F_e, but only 1 quadrature is available.
Lowest $F_{o,I} \to 1$ for $G \to 1$. Ideal $F_{o,I} = 2$ at $G \to \infty$. Why?

<u>Optical amplifier is not special! RX is special: 1 quadrature & detection noise!</u>

Without optical amplifier, true homodyne RX is twice as sensitive as I&Q RX because P_{RX} is not split. But with optical amplifier having $G \to \propto$, output power splitting like in the I&Q RX cannot have an SNR effect. So, behind the amplifier the homodyne RX "must" have the worse sensitivity of the I&Q RX. Amplifier halves homodyne SNR!

Phase-sensitive degenerate parametric optical amplifier passes only 1 quadrature and has ideal $F_{o,I} = 1$ and $F_{o,IQ} = 1/2$ (converts I&Q into more sensitive homodyne).

Zero point fluctuations can explain/replace shot noise

Shot noise can be explained either way:

- Semiclassical theory: Poisson distribution of photoelectrons has one-sided photocurrent power spectral density (PSD) $\boxed{2eI}$.
- Zero point fluctuations interfere with signal and cause shot noise PSD $\boxed{2eI}$.

Let us define field such that power is $P := |\mathbf{E}|^2$. Observation time is $\tau = 1/B_o$.

Zero point fluctuations have mean energy $W = P\tau$ equal to $hf/2$ per mode:

$$\mathbf{E}_0 = (u_1 + ju_2)\underline{\mathbf{e}}_1 e^{j\omega t} \qquad \sigma_{u1}^2 = \sigma_{u2}^2 = hf/(4\tau) \qquad |\underline{\mathbf{e}}_1| = 1$$

Signal field: $\mathbf{E}_S = \sqrt{P_S}\mathbf{e}_1 e^{j\omega t}$ Total field: $\mathbf{E}_S + \mathbf{E}_0$

Expected number of photoelectrons: $n_{S+0} = |\mathbf{E}_S + \mathbf{E}_0|^2 \tau/(hf)$

$$= \left(|\mathbf{E}_S|^2 + 2\,\mathrm{Re}\left(\mathbf{E}_0^+ \mathbf{E}_S\right) + |\mathbf{E}_0|^2 \right)\tau/(hf) \approx \left(P_S + 2u_1\sqrt{P_S}\right)\tau/(hf)$$

Mean: $\langle n_{S+0} \rangle = \frac{P_S \tau}{hf}$ Variance: $\sigma_{nS+0}^2 = \frac{hf}{4\tau} P_S \frac{2^2 \tau^2}{h^2 f^2} = \frac{P_S \tau}{hf} = \langle n_{S+0} \rangle$

$$I = RP = \boxed{\frac{e}{hf}P} \qquad \langle I_{S+0} \rangle = \langle n_{S+0} \rangle \frac{e}{\tau} \qquad \sigma_{IS+0}^2 = \sigma_{nS+0}^2 \frac{e^2}{\tau^2} = \boxed{2e \cdot \boxed{\frac{e}{hf} P_S}} \cdot \boxed{\frac{1}{2\tau}}$$

one-sided electrical bandwidth

I&Q NF derived with zero point fluctuations (1)

$$\mathbf{E}_{RX1} = \left[\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_{11} + ju_{12})\right]\mathbf{e}_1 e^{j\omega t}$$

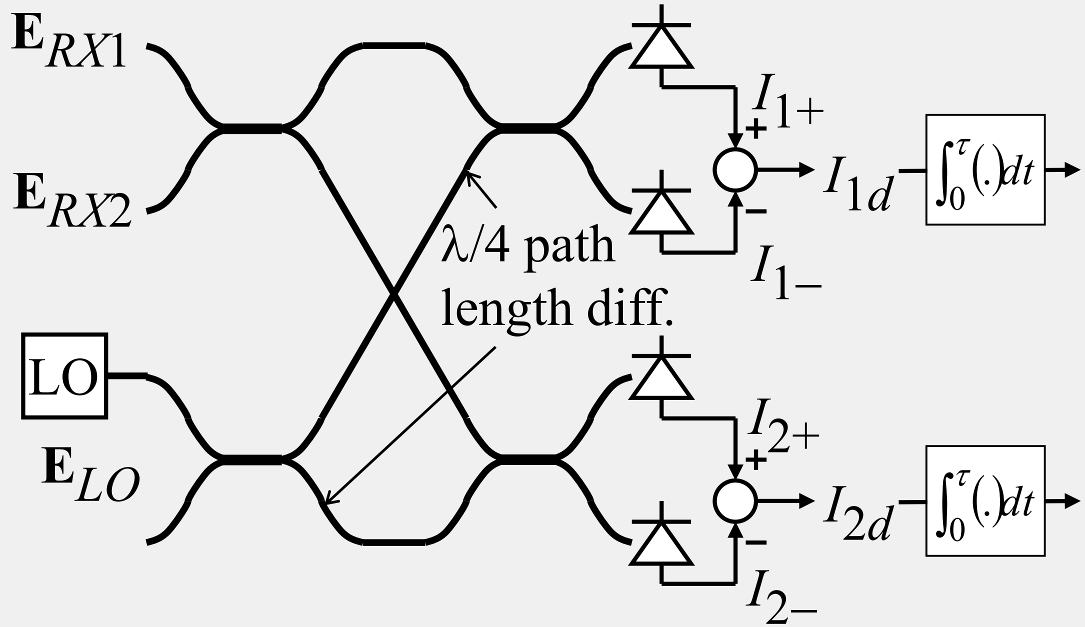
$$\mathbf{E}_{RX2} = (u_{21} + ju_{22})\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2$$

$$w_1 = u_{11} + u_{21} \qquad w_2 = u_{12} - u_{22}$$

$$2\left\langle u_{ij}^2\right\rangle = \left\langle w_k^2\right\rangle = \frac{hf}{2\tau} \qquad \left\langle v_k^2\right\rangle = 1$$



Zero point fluctuations occur at both signal ports. Mean power of zero point fluctuations is neglected for simplicity.

$$I_{1\pm} = R\left|\pm(\mathbf{E}_{RX1} + \mathbf{E}_{RX2})/2 + \mathbf{E}_{LO}/2\right|^2$$

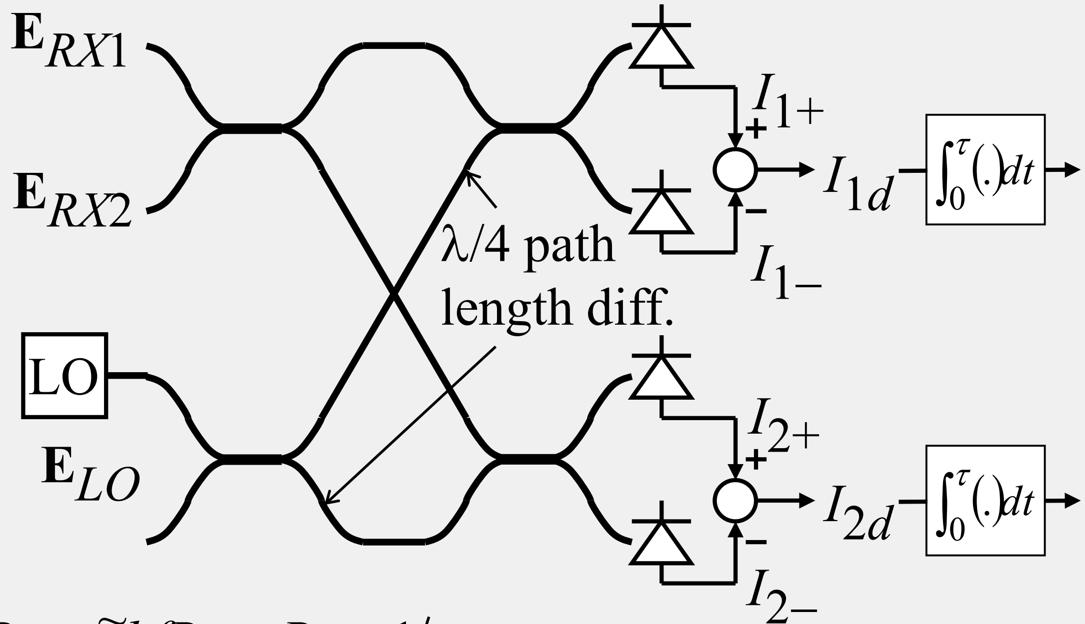
$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

$$I_{2\pm} = R\left|\pm(\mathbf{E}_{RX1} - \mathbf{E}_{RX2})/2 + j\,\mathbf{E}_{LO}/2\right|^2$$

$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}} + P_{LO}\right)$$

I&Q NF derived with zero point fluctuations (2)

The 2 LO ports also carry zero point fluctuations. But these cancel upon subtraction of photocurrents.



$$I_{1d} = I_{1+} - I_{1-}$$
$$= R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}}$$
$$I_{2d} = I_{2+} - I_{2-}$$
$$= R\sqrt{G}\left(v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}}$$

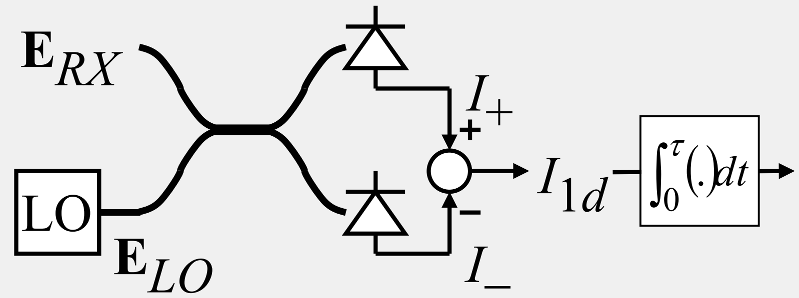
$$\left\langle w_k^2 \right\rangle = \frac{hf}{2\tau} \qquad \left\langle v_k^2 \right\rangle = 1 \qquad P_n = \tilde{\mu} hf B_o \qquad B_o = 1/\tau$$

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO}\left(G P_n/2 + hf/(2\tau)\right)} = \frac{G P_S}{G\tilde{\mu} hf B_o/2 + hf B_o/2} = \frac{P_S \tau}{\left(\tilde{\mu} + 1/G\right) hf/2}$$

$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2} \qquad \frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} = F_{o,IQ} = \tilde{\mu} + 1/G$$

Same result as when derived with shot noise.

Homodyne NF derived with zero point fluctuations



$$\mathbf{E}_{RX} = \left(\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_1 + ju_2)\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2 \qquad \left\langle u_k^2 \right\rangle = \frac{hf}{4\tau} \qquad \left\langle v_k^2 \right\rangle = 1$$

$$I_\pm = R\left|\pm \mathbf{E}_{RX}/\sqrt{2} + \mathbf{E}_{LO}/\sqrt{2}\right|^2$$

$$\approx \frac{R}{2}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

$$I_d = I_+ - I_- = 2R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}}$$

$$SNR_{o,I,out} = \frac{\overline{I_d}^2}{\sigma_{I_d}^2} = \frac{4R^2 P_{LO} G P_S}{4R^2 P_{LO}\left(GP_n/2 + hf/(4\tau)\right)} = \frac{2GP_S}{2G\tilde{\mu} hfB_o/2 + hfB_o/2} = \frac{2P_S\tau}{\left(2\tilde{\mu} + 1/G\right)hf/2}$$

$$SNR_{o,I,in} = \frac{2P_S\tau}{hf/2} \qquad \frac{SNR_{o,I,in}}{SNR_{o,I,out}} = F_{o,I} = 2\tilde{\mu} + 1/G$$

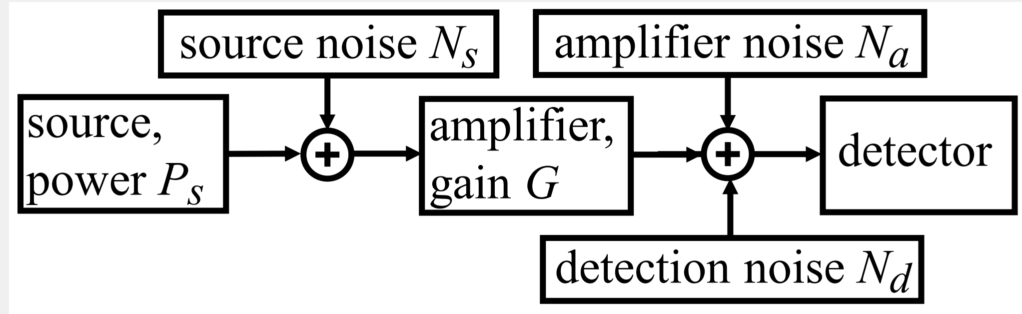
Same result as when derived with shot noise.

Structure of noise figure which fulfills Friis‘ formula

$$SNR_{in}=\frac{P_s}{N_s+N_d}$$

$N_s\sim kTB$?

$N_d\sim hfB$?



$$SNR_{out}=\frac{GP_s}{GN_s+N_d+N_a}$$

$$F=\frac{GN_s+N_d+N_a}{G\left(N_s+N_d\right)}=A+\frac{1-A}{G}+B$$

source noise fraction

$$A=\frac{N_s}{N_s+N_d}$$

added noise fraction

$$B=\frac{N_a}{G\left(N_s+N_d\right)}$$

Device cascade:

$$SNR_{out}=\frac{G_1G_2P_s}{G_1G_2N_s+N_d+G_2N_{a1}+N_{a2}}\qquad F=\frac{G_1G_2N_s+N_d+G_2N_{a1}+N_{a2}}{G_1G_2\left(N_s+N_d\right)}$$

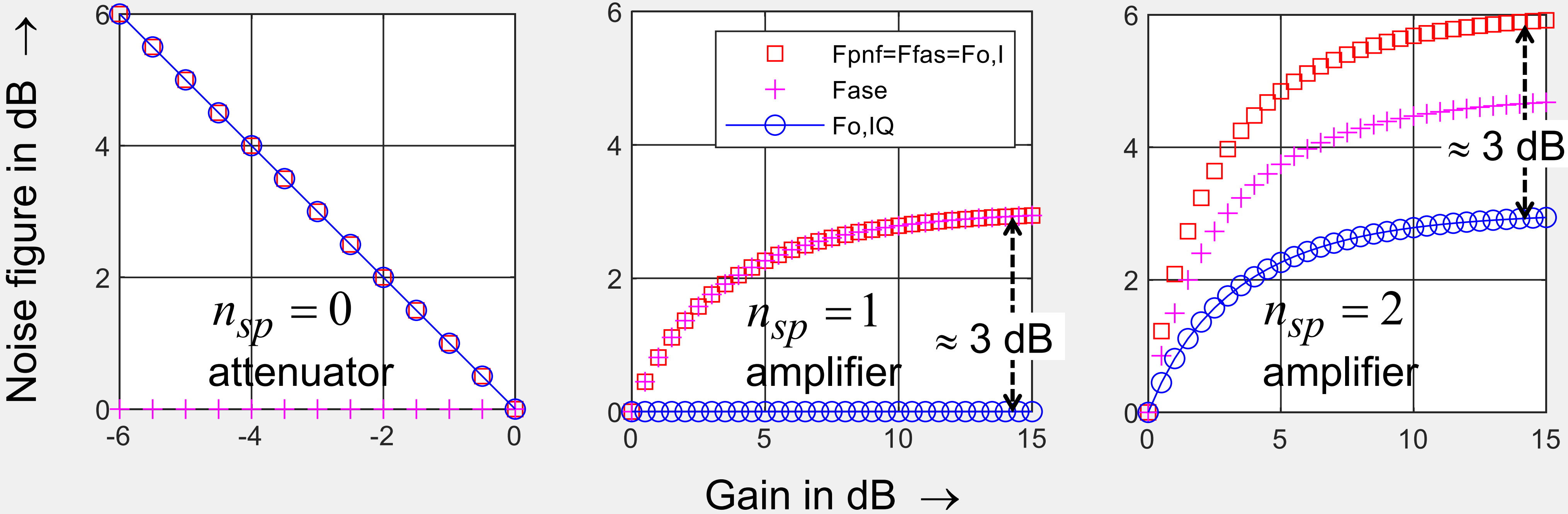
$$\Rightarrow\quad F-1=F_1-1+\frac{F_2-1}{G_1}$$

Complete induction yields Friis' formula:

$$F-1=\sum_{i=1}^{n}\frac{F_i-1}{\prod_{k=1}^{i-1}G_k}$$

It holds for all noise figures which can be written like this, including $\underset{A=1}{F_e,\ F_{ase}}$, $\underset{A=0}{F_{pnf}=F_{fas}=F_{o,I},\ F_{o,IQ}}$!

Optical noise figures [dB] vs. gain [dB]



Only $F_{o,IQ}$ behaves like F_e (1 for amplifier with $n_{sp} = 1$; $1/G$ for attenuator).

2 quadratures, like F_e ☺

$$\boxed{F_{o,IQ} = n_{sp}\left(1-1/G\right)+1/G}$$

1 quadrature ϟ

$$\boxed{F_{pnf} = 2n_{sp}\left(1-1/G\right)+1/G = F_{fas} = F_{o,I}}$$

Is not the SNR degradation factor in any optical receiver!

$$\boxed{F_{ase} = n_{sp}\left(1-1/G\right)+1}$$

Assumes source noise ϟ

Properties of noise figures

Type of noise figure F	SNR degradation factor	Linear	Available quadratures	F of ideal ampl. $G \to \infty$	F of atten., $G < 1$	M of ampl.	Input-referred energy per mode, kT_{ex} or $\tilde{\mu} hf$
F_e	yes	yes	2	1	$1/G$	≥ 0	$kT(F-1)$
$F_{o,IQ} = n_{sp}(1-1/G)+1/G$	yes	yes	2	1	$1/G$	$n_{sp}-1 \geq 0$	$hf(F-1/G)$
$F_{pnf} = F_{fas} = F_{o,I}$ $= 2n_{sp}(1-1/G)+1/G$	yes	not F_{pnf}	1	2	$1/G$	$2n_{sp}-1$ ≥ 1	$hf(F-1/G)/2$
$F_{ase} = 1+n_{sp}(1-1/G)$	no	yes	2	2	1	$n_{sp} \geq 1$	$hf(F-1)$

Only $F_{o,IQ}$ matches conceptually with F_e!

For lowest NF of a cascade, order amplifiers according to ascending noise measure M.

$$M = \frac{F-1}{1-1/G}$$

Ideal optical amplifier noise figure at large gain is ... ?

$$\text{ideal optical NF} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}}$$

(according to the foregoing)

quadratures	1	2
optical amplifier	phase-sensitive	phase-insensitive
optical receiver	direct detection (or homodyne)	I&Q, or heterodyne with image rejection

Common answer since mid 1990ies:

$$F_{pnf} = 2 = F_{fas} = F_{o,I}$$

But with the same logic one could answer: $F_{o,IQ} = 1/2$

Other cases are considered as special.

It makes most sense to pair amplifiers and receivers with same number of available quadratures:

optical amplifier	phase-sensitive	phase-insensitive
optical receiver	homodyne (or direct detection)	I&Q, or heterodyne with image rejection
Nonlinear! Can it yield a NF?	$F_{o,I} = 1$	$F_{o,IQ} = 1$ (like $F_e = 1$)

By far most frequent optical + electrical scenario <u>today</u>!

User must provide phase reference! RX can also contain phase-sensitive amplifier!

2 unequal NF for 1 scenario? 1 NF for 2 unequal scenarios?

One cannot say one NF (F_e) is for electrical detectors and another (F_{pnf}) is for quantum detectors (photodiodes), because one might become able to build both detector types for the same f (low THz region?): This would oppose unequal NF for same usage of same amplifier at same f! NF must be detector-independent!

The term "noise figure" without additions suggests the properties of F_e, i.e. SNR degradation factor in linear system with 2 quadratures (and preferably Gaussian noise).

$\Rightarrow$ Term "optical noise figure" seems fit only for $F_{o,IQ} = \tilde{\mu} + 1/G$.

To avoid misinterpretation, $F_{pnf} = 2\tilde{\mu} + 1/G$ could be called "high-power optical χ^2 (chi-square) noise estimator", "photoelectron number fluctuation indicator", ...

Likewise, $F_{o,I}\ (= F_{fas} (= F_{pnf}))$ can be called "optical 1-quadrature NF" (= in-phase).

If SNR is defined with only in-phase noise then the electrical 1-quadrature NF $F_{e,I}$ equals F_e. I have combined $F_{e,I}$ with $F_{o,I}$ to form a 1-quadrature NF.

Result is equivalent to the unified F_{fas}. But number of quadratures in F_{fas} is not given and one is left to assume that in the electrical domain F_{fas} is for 2 quadratures. $1 \neq 2$!

An interpretation difference is that in F_{fas} added thermal noise is considered not separately, but as caused by spontaneous emission (set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$). In a phase-sensitive amplifier, ideal $F_{o,I} = F_{fas} = 1$.

SNR in the presence of thermal and optical noises

To derive a consistent unified NF (**I&Q** !) we add noises of F_e and $F_{o,IQ}$ for all f.

Optical and electrical gains G are identical because they manifest at the same f.

Total thermal noise in bandwidth B_o at the amplifier output is $GF_e kTB_o$. Half of this is in phase with the signal. In the coherent I&Q RX it appears multiplied with $R^2 P_{LO}$, like the amplified signal power GP_S. The corresponding variance σ_e^2 is added.

$$SNR_{IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2}$$

Detector type does not matter, as long as it is usable in linear I&Q receiver:

$$= \frac{R^2 P_{LO} G P_S}{R^2 P_{LO} GF_e kTB_o/2 + R^2 P_{LO} \tilde{\mu} GhfB_o/2 + eRP_{LO}B_e}$$

Powers in I&Q receiver with quantum detectors

$$= \frac{GP_S}{GF_e kTB_o/2 + \tilde{\mu} GhfB_o/2 + hfB_e}$$

Powers in electrical I&Q receiver

$$= \frac{P_S \tau}{F_e kT/2 + F_{o,IQ} hf/2} = \frac{P_S \tau}{k(T + T_{ex})/2 + (\tilde{\mu} + 1/G)hf/2}$$

- Thermal source noise
- Thermal amplifier noise
- Spontaneous emission field noise in amplifier
- Shot noise in detector

I&Q noise figure from electrical to optical frequencies

$$SNR_{IQ,out} = \frac{P_S \tau}{F_e kT/2 + F_{o,IQ} hf/2} = \frac{P_S \tau}{k(T+T_{ex})/2 + (\tilde{\mu}+1/G)hf/2}$$

Linear!

Pure Gaussian noises!

$$SNR_{IQ,in} = \frac{P_S \tau}{kT/2 + hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{IQ,in}}{SNR_{IQ,out}} = \boxed{F_{IQ} = \frac{F_e kT + F_{o,IQ} hf}{kT+hf} = \frac{k(T+T_{ex}) + (\tilde{\mu}+1/G)hf}{kT+hf}}$$

2 available quadratures!

$$= A + (1-A)/G + \left(A T_{ex}/T + (1-A)\tilde{\mu}\right)$$

$$\boxed{A = kT/(kT+hf)}$$

Fulfills Friis' formula!

Measured F_{IQ} is just observed SNR degradation in linear system with 2 quadratures.

In amplifier, F_e, $F_{o,IQ}$ may not be known. Anyway, $kT_{ex} + \tilde{\mu}hf$ is total added noise.

In attenuator, clear separation yields the correct result: $G<1,\ T_{ex} = T(1/G-1)$,

$n_{sp}=0,\ \tilde{\mu}=0 \quad \Rightarrow \quad F_{IQ} = 1/G = F_e = F_{o,IQ}$

https://ieeexplore.ieee.org/document/9783564 = 66 GHz @ 4 K

At low f: $F_{IQ} \to F_e$. At high f: $F_{IQ} \to F_{o,IQ}$.

At 9300 / 1350 / 300 / 77 / 4 K, equal $kT = hf$ is at f = 194 / 28 / 6 / 1.6 / 0.08 THz.

SNR with 1-quadrature noises and homodyne receiver

To derive a unified NF for only 1 quadrature we add noises of F_e and $F_{o,I}$ for all f.

No power splitting $\Rightarrow$ $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ must be multiplied by **2** compared to $F_{o,IQ}$ calculation. Total thermal noise in bandwidth B_o at amplifier output is GF_ekTB_o. Half of this is in phase with the signal. In the coherent 1-quadrature (homodyne) RX it appears multiplied with $4R^2P_{LO}$, like the amplified signal power GP_S.

$$\boxed{F_{e,I} = F_e}$$

$$SNR_{I,out} = \frac{4\overline{I_{1d}}^2}{4\sigma_e^2 + 4\sigma_{I_{1d}}^2 + 2\sigma_{I_{1s}}^2}$$

(Quantities found in I&Q RX are multiplied here by **2**·**2** or **2**.)

$$= \frac{4R^2P_{LO}GP_S}{4R^2P_{LO}GF_ekTB_o/2 + 4R^2P_{LO}\tilde{\mu}GhfB_o/2 + 2eRP_{LO}B_e}$$

Powers in homodyne receiver with quantum detectors

$$= \frac{2GP_S}{2GF_ekTB_o/2 + 2\tilde{\mu}GhfB_o/2 + hfB_e}$$

$$= \frac{2P_S\tau}{F_ekT + F_{o,I}hf/2} = \frac{2P_S\tau}{k(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}$$

Thermal source noise
Thermal amplifier noise
Spontaneous emission field noise in amplifier
Shot noise in detector

1-quadrature / homodyne unified noise figure

$$SNR_{I,out} = \frac{2P_S\tau}{F_e kT + F_{o,I} hf/2} = \frac{2P_S\tau}{k(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}$$

$$SNR_{I,in} = \frac{2P_S\tau}{kT + hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{I,in}}{SNR_{I,out}} = F_I = \frac{F_e kT + F_{o,I}\, hf/2}{kT + hf/2} = \frac{k(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}{kT + hf/2}$$

$$= A_I + (1-A_I)/G + \left(A_I\, T_{ex}/T + (1-A_I)2\tilde{\mu}\right)$$ $\boxed{A_I = kT/(kT+hf/2)} \neq A$

$F_{o,I} \neq F_{o,IQ}$ because there is detection noise!

$F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!

(set $T_{ex}=0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$)

1-quadrature / homodyne F_I equals F_{fas} (except interpretation difference)!

In definition of F_{fas}, number of quadratures was not discussed. F_{fas} is intended to be identical with the normal electrical F_e, which is understood to be for 2 available quadratures. So, one is left to assume that F_{fas} has 2 quadratures in the electrical and 1 quadrature in the optical domain. But that is contradictory, impossible!

1-quadrature / homodyne unified noise figure

$$SNR_{I,out} = \frac{2P_S\tau}{F_e kT + F_{o,I} hf/2} = \frac{2P_S\tau}{k\left(T+T_{ex}\right)+\left(2\tilde{\mu}+1/G\right)hf/2}$$

$$SNR_{I,in} = \frac{2P_S\tau}{kT + hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{I,in}}{SNR_{I,out}} = F_I = \frac{F_e kT + F_{o,I}\,hf/2}{kT+hf/2} = \frac{k\left(T+T_{ex}\right)+\left(2\tilde{\mu}+1/G\right)hf/2}{kT+hf/2}$$

$$= A_I + \left(1-A_I\right)/G + \left(A_I\,T_{ex}/T + \left(1-A_I\right)2\tilde{\mu}\right) \qquad \boxed{A_I = kT/\left(kT+hf/2\right)} \neq A$$

$F_{o,I} \neq F_{o,IQ}$ because there is detection noise!

$F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!

1-quadrature / homodyne F_I equals F_{fas} (except interpretation difference)!

Attenuator: I simply say

$T_{ex} = T\left(1/G-1\right),\ n_{sp} = 0 = \tilde{\mu}\,,$

$F_I = 1/G = F_{o,I} = F_e \quad \left(= F_{e,I}\right).$

Attenuator: To get $F_{fas} = 1/G \ \left(= F_I\right)$ I find I must set $T_{ex}=0,\ n_{sp} = -kT/\left(hf\right),\ \tilde{\mu} = n_{sp}\left(1-1/G\right).$

$f \to \{\infty, 0\} \ \Rightarrow\ n_{sp} \to \{0, -\infty\},\ \tilde{\mu} \to \{0, \infty\}$!

Summary

- All prior optical and unified NF F_{pnf}, F_{fas}, F_{ase} are in conflict with electrical NF F_e.
- A „noise figure“ without special name is expected to be the SNR degradation factor in a linear system with 2 available quadratures (and Gaussian noise?!), like F_e.
- The only optical NF which fulfills this is the optical I&Q NF $F_{o,IQ}$. It is ≥ 1, like F_e.
- Coherent I&Q receivers are linear field sensors. They linearize the quadratic field behavior of photodiodes. Heterodyne with image rejection is also fine.
- At high gain, $F_{o,IQ} \approx F_{pnf}/2$, i.e.≈ 3 dB less when expressed in dB.
- Electrical and optical I&Q NF are limit cases of only one unified NF F_{IQ} for all f.
  Quantum noise / F_{IQ} plays a role in today‘s electronics at low $T = 4\ \mathrm{K}$.
- The in-phase equivalent of $F_{o,IQ}$ is $F_{o,I}$, a limit case of the unified F_{fas}. So, F_{fas} is a 1-quadrature NF and its other limit is F_e for 1 quadrature, not the expected 2.
- Information conveyed by the full F_{pnf} of a specific receiver can be obtained, more accurately, from $F_{o,IQ}$ (pure Gaussian noise).
- Optical amplifier adds Gaussian I&Q field noise (wave aspect).
  Photodetection adds shot noise (particle aspect).